

**World Logic Day in Aveiro Department of Mathematics, University of Aveiro**  
**Part of 3rd WORLD LOGIC DAY**

On behalf of the celebration of UNESCO 3rd World Logics Day, we organised at Mathematics Department of University of Aveiro the workshop "World Logic day at Aveiro"(due the pandemic situation, it was proceeded by zoom) constituted by a three interesting talks that were enriched with deep discussion from a the audience.

**LOCAL ORGANISERS**

**MANUEL A. MARTINS**

**ALEXANDRE MADEIRA**

**Invited Talk**

[Rui Soares Barbosa](#), INL – International Iberian Nanotechnology Laboratory, Portugal

**Partial Boolean algebras: the logic of contextually**

(joint work with Samson Abramsky)

**ABSTRACT:** Contextually is a key signature of quantum non-classicality, which has been shown to play a central role in enabling quantum computational advantage. Kochen and Specker's seminal work on contextuality contains elements of a logical flavour that have largely been overlooked in subsequent literature on the topic. In particular, it introduced the notion of partial Boolean algebra, which provides a natural (algebraic) logical setting for studying contextual systems. It contrasts with traditional quantum logic à la Birkhoff and von Neumann in that operations such as conjunction and disjunction are partial, being only defined in the domain where they are physically meaningful. In the key example of the projectors on a Hilbert space, the operations are only defined for commuting projectors, which correspond to properties of a quantum system that are commensurable, i.e. can be measured simultaneously.

In this talk, we will give an introduction to partial Boolean algebras and discuss various topics arising in our recent work, including:

- the formulation of contextuality properties in this setting, including Kochen–Specker paradoxes, logically contradictory statements that are validated by a partial Boolean algebra;
- the Logical Exclusivity Principle and its relation to Probabilistic Exclusivity widely studied in the quantum foundations literature as a step to closing in on the set of quantum-realizable correlations;
- work towards a logical presentation of the Hilbert space tensor product, using logical exclusivity to capture some of its salient quantum features.

A central role in this is played by a universal construction that freely extends the commensurability relation on a partial Boolean algebra. This is given through a concrete inductive presentation by generators and relations.

This is joint work with Samson Abramsky, and can be found at [arXiv:2011.03064](https://arxiv.org/abs/2011.03064) [quant-ph].

**Contributed Talks**

[Eros Martinelli](#), UA, Aveiro, PORTUGAL

**Injective Hulls of Quantale-Enriched Multicategories**

**ABSTRACT:** It is well known that there is a connection between the Dedekind-MacNeille completion of an ordered set and its injective hull. Namely, given a poset  $X$ , its injective hull (with respect to embeddings) is the Dedekind-MacNeille completion of  $X$ . This construction can be generalized to the realm of quantale-enriched categories where, in a similar way, one can build injective hulls as algebras for the monad that arises from the Isbell adjunction. In this talk we study this problem in the realm of quantale-enriched multicategories, a generalization of promonoidal categories. This kind of categories naturally appear when one wants to "mix" cocompleteness with monoidal completions. The classical example is the construction of the free quantale  $Q$  starting from an ordered set  $X$ . First one has to generate an ordered monoid out of  $X$  by taking finite lists, then one has to add all possible suprema in order to make it complete. In this way, every element of  $Q$  is a suprema of lists of element of  $X$ ; this is a particular example of "monoidal" colimit, which naturally arises when one study colimits for quantale-enriched multicategories. Unfortunately, the situation is not so smooth as in the "classical" case. In order to be able to construct injective hulls, we have to make a detour to the category of the so-called quantum B-algebras, representable promonoidal categories. For this category we will be able to mimic all the constructions done for quantale-enriched

categories and build injective hulls as algebras for a lax-monoidal monad which resembles the one induced by the Isbell adjunction. Luckily, the restriction to quantum B-algebras does not prevent us from constructing an injective hull for every multicategory, by embedding every multicategory in a (suitable) quantum B-algebra we will provide the injective hull we were searching for.

Leandro Gomes, INESC TEC, U. MINHO, Portugal

### **Semantics and dynamic logics for fuzzy programs**

(joint work with Alexandre Madeira and Luís Soares Barbosa)

**ABSTRACT:** Fuzzy programming languages emerged to describe systems that reason about information which cannot be evaluated in simple terms of “true” and “false”. Typical examples of these languages find its utility in distinct application domains, such as medical diagnosis [3] and robotics [1]. The syntax of these programming languages include variables storing information as fuzzy sets, and a set of conditional rules to describe the behaviour of the system. Such rules, which syntactically are just if-then-else and switch-case statements do not behave nondeterministically, as it happens in the more classic scenario, presenting instead a parallel behaviour inherent to their execution, due to the nature of the fuzzy information that is evaluated. We introduce a family of  $\lambda$ -free dynamic logics for reasoning about fuzzy conditionals, with a semantics where programs are interpreted as fuzzy binary multirelations. Such mathematical concept generalises binary multirelations [2] to model an execution from one state to a set of states in parallel, with (possible) different weights associated with each branch of execution. The method is parametric on an arbitrary complete right residuated lattice, offering a suitable truth space to deal with fuzzy information.

References:

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- [2] I. Rewitzky, Binary multirelations, In Harrie C. M. de Swart, Ewa Orlowska, Gunther Schmidt, and Marc Roubens, editors, *Theory and Applications of Relational Structures as Knowledge Instruments*, COST Action 274, TARSKI, Revised Papers, volume 2929 of *Lecture Notes in Computer Science*, pp. 256–271, Springer, 2003.
- [3] T. Vetterlein H. Mandl and K. Adlassnig, Fuzzy arden syntax: A fuzzy programming language for medicine, *Artificial Intelligence in Medicine*, 49(1), pp. 1–10, 2010.

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